

# On meson melting in the quark medium

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## Abstract

We consider a heavy quark-antiquark ( $q\bar{q}$ ) pair as a heavy meson in the medium composed of light quarks and gluons. By using the AdS/CFT correspondence, the properties of this system are investigated. In particular, we study the inter-quark distance and it is shown that the mechanism of melting in the quark-gluon plasma and in the hadronic phase are the same. It is found that by considering finite coupling corrections, the inter-quark distance of a heavy meson decreases. As a result a heavy meson like  $J/\psi$  will melt at higher temperatures. By considering rotating heavy mesons, we discuss melting of excited states like  $\chi_c$  and  $\psi'$ .

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## 1 Introduction

The experiments of Relativistic Heavy Ion Collisions (RHIC) have produced a strongly-coupled quark–gluon plasma (QGP)(see review [1]). At a qualitative level, the data indicate that the QGP produced at the LHC is comparably strongly-coupled [2]. The QGP that is created at the LHC is expected to be better approximated as conformal than is the case at RHIC [1]. There are no known quantitative methods to study strong coupling phenomena in QCD which are not visible in perturbation theory (except by lattice simulations). A new method for studying different aspects of QGP is the *AdS/CFT* correspondence [1, 3, 4, 5, 6]. This method has yielded many important insights into the dynamics of strongly-coupled gauge theories. It has been used to investigate hydrodynamical transport quantities in various interesting strongly-coupled gauge theories where perturbation theory is not applicable. Methods based on *AdS/CFT* relate gravity in  $AdS_5$  space to the conformal field theory on the four-dimensional boundary [5]. It was shown that an *AdS* space with a black brane is dual to a conformal field theory at finite temperature [6].

In heavy ion collisions at the LHC, heavy-quark related observables are becoming increasingly important [7]. In these collisions, one of the main experimental signatures of QGP formation is melting of heavy mesons in the medium [8]. In this paper we study melting of a heavy meson from AdS/CFT. We consider a heavy quark-antiquark ( $q\bar{q}$ ) pair in the medium composed of light quarks and gluons as a heavy meson [10]. By using the AdS/CFT correspondence, the properties of this system are investigated.

We will consider the holographic QCD in the quark medium which was studied in [11, 12]. It was shown that at the high temperature, the gravity dual to the quark-gluon plasma is the Reissner-Nordstrom AdS ( $RNAdS$ ) black hole and at the low temperature, the dual geometry corresponding to the hadronic phase is the thermal charged AdS ( $tcAdS$ ) space. It was also found that ( $tcAdS$ ) space can be obtained by taking zero mass limit of the  $RNAdS$

black hole [11]. The confinement/deconfinement phase transition in the quark medium was discussed in [12] and an influence of matters on the deconfinement temperature,  $T_c$  was investigated. Using a different normalization for the bulk gauge field, it was shown that the critical baryonic chemical potential becomes  $1100 MeV$  which is comparable to the QCD result [13].

Melting of a heavy meson is investigated in [13] and it is found that in the quark-gluon plasma the dissociation length <sup>1</sup> becomes shorter as the chemical potential increases. On the contrary, in the hadronic phase the dissociation length becomes larger as the chemical potential increases. In the next section, we argue that this conclusion is not valid and explicitly show that in the hadronic phase (low temperature), the dissociation length decreases by increasing the chemical potential. We conclude that the melting mechanism in the quark-gluon plasma and in the hadronic phase are the same *i.e.* the interaction between heavy quarks is screened by the light quarks.

In third section, we consider higher derivative corrections *i.e.*  $\mathcal{R}^2$  which corresponds to the finite coupling corrections on the *static* quark-antiquark system in the hot plasma. To study  $\mathcal{R}^2$  corrections, static heavy meson in Gauss-Bonnet background has been considered. It is shown that at a given energy, the increase in the  $\lambda_{GB}$  leads to a decrease in the dissociation length. This confirms the results of [20] *i.e.* considering the higher derivative corrections in the gravity background leads to a decrease in the dissociation length of heavy meson in the boundary gauge theory. One may conclude that the heavy meson melting at finite coupling occurs at higher temperature which confirms that heavy mesons like quarkonia need higher temperature to melt. It was explored in [8] that  $J/\psi$ s can be used as a confinement/deconfinement probe. As a result,  $J/\psi$  will melt at higher temperature than the excited states like  $\chi_c$  and  $\psi'$  [9].

To study melting of excited states in the quark medium, we consider a rotating heavy meson. It is shown that the dissociation length increases as the spin increases which confirms that the excited states will melt at lower temperature than the ground states. We also investigate the effect of increasing flavor quarks on the dissociation length. It is shown that as the flavor quarks increase, the dissociation length decreases. We summarize the results in the conclusion section.

## 2 Holographic melting in the quark medium

In this section we will give a brief review of [13]. The Euclidean action describing the five-dimensional asymptotic  $AdS$  space with the gauge field is given by

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<sup>1</sup>One may call it screening length.

$$S = \int d^5x \sqrt{G} \left( \frac{1}{2\kappa^2} (-\mathcal{R} + 2\Lambda) + \frac{1}{4g^2} F_{MN} F^{MN} \right), \quad (2.1)$$

where  $\kappa^2$  is proportional to the five-dimensional Newton constant and  $g^2$  is a five-dimensional gauge coupling constant. The cosmological constant is given by  $\Lambda = \frac{-6}{R^2}$ , where  $R$  is the radius of the  $AdS$  space. It was pointed out that the quark-gluon plasma and hadronic phase could be described by the  $tcAdS$  and the  $RNAdS$  black hole, respectively [13]. These solutions can be considered as follows

$$ds^2 = \frac{R^2}{z^2} \left( f(z) dt^2 + d\vec{x}^2 + \frac{1}{f(z)} dz^2 \right), \quad (2.2)$$

where the function  $f(z)$  for  $RNAdS$  black hole is given by

$$f(z) = 1 - mz^4 + q^2 z^6, \quad (2.3)$$

and in the case of  $tcAdS$  space

$$f(z) = 1 + q^2 z^6. \quad (2.4)$$

In these coordinates,  $z$  denotes the radial coordinate of the black hole geometry and  $t, \vec{x}$  label the directions along the boundary at the spatial infinity. The event horizon is located at  $f(z_h) = 0$  where  $z_h = z_+$  is the largest root and it can be found by solving this equation. The boundary is located at  $z = 0$  and the geometry is asymptotically  $AdS$  with radius  $R$ .

The parameters  $m$  and  $q$  are the black hole mass and charge, respectively. The time-component of the bulk gauge field is  $A_t(z) = i(2\pi^2\mu - Q z^2)$  where  $\mu$  and  $Q$  are related to the chemical potential and quark number density in the dual gauge theory. Regarding the Dirichlet boundary condition at the horizon,  $A_t(z_+) = 0$ , one finds  $Q = \frac{2\pi^2\mu}{z_+}$ . The black hole charge  $q$  and the quark number density  $Q$  also are related to each other by this equation

$$Q = \sqrt{\frac{3g^2 R^2}{2\kappa^2}} q. \quad (2.5)$$

From now on, we investigate the holographic melting of heavy mesons in two phase of the quark medium, the quark-gluon plasma and the hadronic phase, respectively. The holographic melting of heavy mesons at finite temperature in a chiral and confining string dual is discussed in [16]. It is argued in [17] that the heavy quark and anti-quark potential becomes zero at a separation  $r_d$  and for large separation, the dominant configuration corresponds to two straight strings and the string melts. Then, one should study the binding energy of heavy meson  $V_b(r)$  when it goes to zero<sup>2</sup>. This phenomena happens at special length  $r_d$

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<sup>2</sup>This is the large  $N_c$  and large 't Hooft coupling definition of heavy quark potential. A careful discussion of the corrections to this definition was done in [14].

which is obtained from  $V_b(r_d) = 0$ . One may call  $r_d$  as the screening length of heavy meson [18]. The screening length  $L_s$  of a heavy quark-antiquark pair in strongly coupled gauge theory plasmas flowing at velocity  $v$  is studied in [19]. There, screening length is defined so that for  $L > L_s$  no extremal world-sheet exists which binds the quark-antiquark pair. Regarding the conventions of [13], we call  $r_d$  dissociation length.

One finds from the standard calculations of [13] that the inter-quark distance is given by

$$r = 2 \int_0^{z_0} dz z^2 \frac{\sqrt{f(z_0)}}{\sqrt{f(z)}} \frac{1}{\sqrt{f(z)z_0^4 - f(z_0)z^4}}. \quad (2.6)$$

The inter-quark distance in AdS-Schwarzschild black hole is investigated in [17]. It is shown that for long strings <sup>3</sup>

$$\frac{r}{2} = A c \left( \frac{1}{a} - \frac{1}{5a^5} - \frac{1}{10a^9} - \dots \right) \quad (2.7)$$

where  $A$  is a constant and

$$c = \sqrt{2} \mathbf{E}(1/\sqrt{2}) - \frac{1}{\sqrt{2}} \mathbf{K}(1/\sqrt{2}). \quad (2.8)$$

The inter-quark distance of a static meson in the quark-gluon phase and in two special limits can be studied as follows

- $z_0 \simeq z_+$

This limit corresponds to large strings which means that the U-shaped string touches the horizon. Then one should assume  $f(z_0) = \epsilon \simeq 0$  and the equation for  $r$  becomes

$$r = \frac{2\sqrt{\epsilon}}{z_0^2} \int_0^{z_0} dz \frac{z^2}{f(z)}. \quad (2.9)$$

It is clearly seen that there is a singularity which is reasonable. In the case of extremal  $RNAdS$  black hole, one finds that  $f(z) \simeq (z - z_0)^2$ .

- $z_0 \simeq 0$

Now we consider  $z_0 \rightarrow \epsilon \simeq 0$ . It corresponds to the short strings. In this case  $f(\epsilon) \simeq 1$  and one finds that (2.6) becomes

$$r/2 = \int_0^\epsilon \frac{dz}{\sqrt{-q^2 z^6 + m z^4 - 1}} \quad (2.10)$$

where the solution is expressed in terms of Incomplete Elliptic functions. See appendix A.

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<sup>3</sup>We would like to thank S. Sheikh-Jabbari for discussion on this expansion.

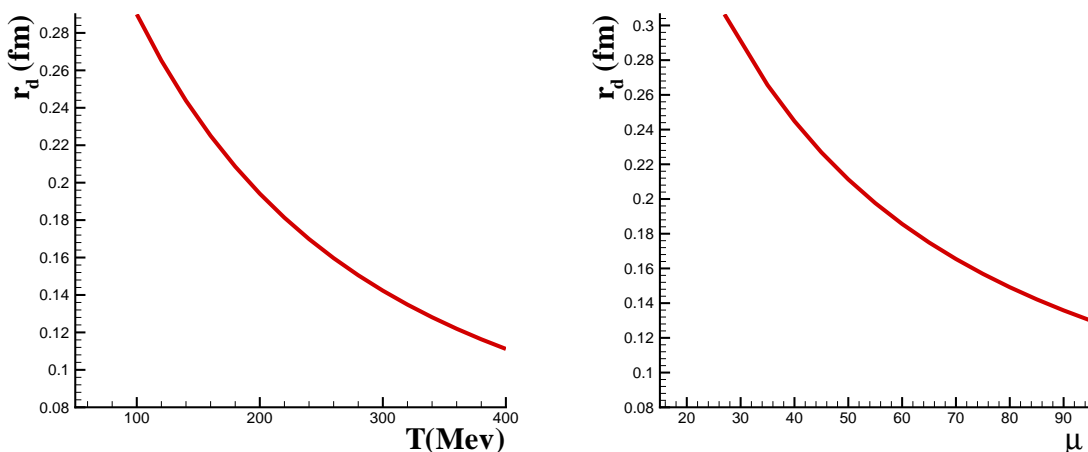


Figure 1: Left: The inter-quark distance versus the temperature of the quark-gluon plasma at fixed chemical potential  $\mu = 30 \text{ MeV}$  and  $n = 1$ . Notice that the temperature must be larger than  $91.2 \text{ MeV}$ . Right: The inter-quark distance versus the chemical potential of the quark-gluon plasma at fixed temperature  $T = 100 \text{ MeV}$  and  $n = 1$ . Notice that  $\mu > 26.8 \text{ MeV}$ .

One finds from the standard calculations of [13] that the binding energy is

$$V_b = \frac{R^2}{\pi\alpha'} \left( \int_0^{z_0} dz z^{-2} \frac{\sqrt{f(z)}}{\sqrt{f(z) - \frac{f(z_0)z^4}{z_0^4}}} - \int_0^{z_1} dz z^{-2} \right) \quad (2.11)$$

In the above equation, the second term is the energy for two free heavy quarks.<sup>4</sup> The nearest point of U-shaped string to the horizon is shown by  $z_0$ . Notice that  $z_1 > z_0$  and in the quark-gluon plasma  $z_1$  is the outer horizon of the  $RNAdS$  black hole  $z_+$  which is given by

$$z_+ = \frac{3g^2 R^2}{8\pi^4 \kappa^2 \mu^2} \left( \sqrt{\pi^2 T^2 + \frac{16\pi^4 \kappa^2 \mu^2}{3g^2 R^2}} - \pi T \right). \quad (2.12)$$

where  $T$  is the temperature of the quark-gluon plasma. In the hadronic phase,  $z_1$  is the IR cut-off  $z_{IR}$  [13].

We repeat the calculations of [13] and plot the  $r_d$  depending on the temperature and the chemical potential in Fig. 1. This figure corresponds to Fig. 3. in [13]. These plots imply that the  $r_d$  becomes short as the temperature and chemical potential of quark-gluon plasma increases. In the hadronic phase, one should notice that in (2.6) and (2.11),  $f(z)$  is given by (2.4). It would be interesting to investigate the behavior of  $r_d$  in terms of the chemical

<sup>4</sup>The main mistake in the calculations of [13] comes from this term. This mistake has an important effect in the hadronic phase when we study the dissociation length versus the chemical potential in Fig. 2.

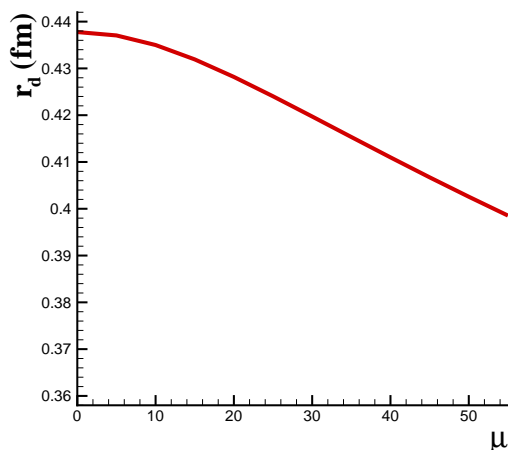


Figure 2: The dissociation length versus the chemical potential in the hadronic phase and  $n = 1$ . This phase exists at the zero temperature then we do not have the temperature dependency of the  $r_d$ .

potential. We show the result in Fig. 2. It is clearly seen that by increasing the chemical potential, the dissociation length becomes shorter. On the contrary, the Fig. 5 of [13] shows that the dissociation length becomes larger. We found that the reason of this wrong result is the mistake in the free energy of two quarks in (2.11).<sup>5</sup> As a result one concludes similar physics for melting of heavy mesons in the quark-gluon plasma and hadronic phase of QCD. In the QGP phase, the interaction between heavy quarks is screened by the light quarks so that the dissociation length of the heavy meson decreases as the temperature or the light quark chemical potential becomes larger.

It is clearly seen that the shape of  $r_d$  versus the chemical potential in the quark-gluon plasma and in the hadronic phase are not the same. Compared with the Fig. 2, the decrease in dissociation length in Fig. 1 is more noticeable. One possible reason is that in the hadronic phase, there is no temperature dependence in the holographic QCD model and one considers the zero temperature case only. As a result, the role of temperature in the melting of heavy meson is absent in the hadronic phase.

We introduce  $N_f = nN_c$  where  $N_f$  and  $N_c$  are the number of flavors and color fields, respectively. We are going to study how  $r_d$  changes when flavor number of quarks increases [13]. We plot the binding energy of heavy meson versus the inter-quark distance in the Fig. 3. In the left and right plot of this figure, we have considered the quark-gluon plasma and hadronic phase of QCD. It is clearly seen that in each phase of QCD,  $r_d$  becomes shorter as the flavor number becomes larger. Compared with the Fig. 4. of [13], one finds that in the

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<sup>5</sup>We would like to thank Chanyong Park for discussion on this point.

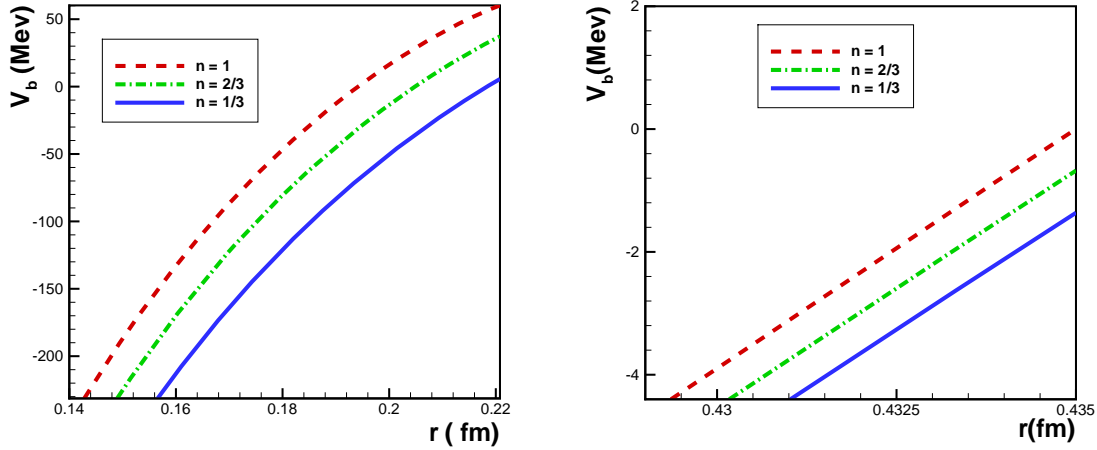


Figure 3: Left: The binding energy versus the inter-quark distance in the quark-gluon plasma at  $(\mu, T) = (30 \text{ MeV}, 200 \text{ MeV})$ . Right: The binding energy versus the inter-quark distance in the hadronic phase and  $(\mu, T) = (10 \text{ MeV}, 0 \text{ MeV})$ .

hadronic phase as the flavor number increases  $r_d$  decreases. This implies that as the flavor number increases, the heavy meson melts easier. The meaning of the melting in the hadronic phase was discussed in [13]. It implies that the heavy meson is broken into the light mesons which forms from two heavy-light quarks bound states.

### 3 Holographic melting at finite coupling

In this section, we investigate the effect of the curvature-squared corrections on the screening length. These corrections are related to the finite-coupling correction in the corresponding gauge theory. It was shown that these corrections affect the dissociation length and it was argued that the increase of the coupling of the corrections decreases the  $r_d$  [20]. Now we continue our study to the case of a heavy meson in the quark medium.

To study effects of the finite 't Hooft coupling, we consider the Reissner-Nordström-AdS black brane solution in Gauss-Bonnet gravity [21]. The following action term in 5 dimensions describes the Gauss-Bonnet term which should be considered in (2.1) as

$$S_{\mathcal{R}^2} = \frac{R^2 \lambda}{4\kappa^2} (\mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}), \quad (3.1)$$

where  $\mathcal{R}_{\mu\nu\rho\sigma}$ ,  $\mathcal{R}_{\mu\nu}$  and  $\mathcal{R}$  are the Riemann curvature tensor, Ricci tensor, and the Ricci scalar, respectively. The Gauss-Bonnet coupling constant is  $\lambda$ . The charged black brane solution in 5-dim is given by

$$ds^2 = \frac{R^2}{z^2} \left( N^2 f(z) dt^2 + d\vec{x}^2 + \frac{1}{f(z)} dz^2 \right), \quad (3.2)$$



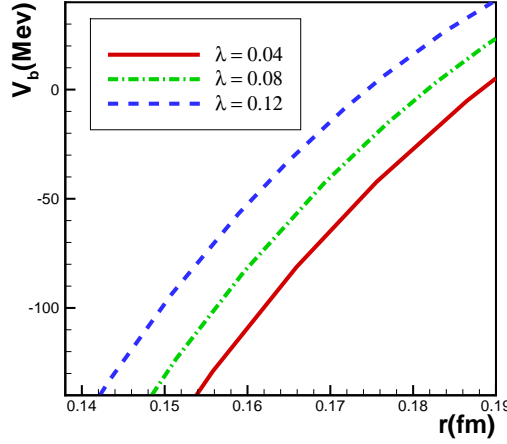


Figure 4: The binding energy of the heavy meson versus the inter-quark distance for different values of  $\lambda = 0.04, 0.08$  and  $0.12$ . Graph is made for  $n = 1$  and  $(\mu, T) = (30 \text{ MeV}, 200 \text{ MeV})$ .

where

$$f(z) = \frac{1}{2\lambda} \left( 1 - \sqrt{1 - 4\lambda(1 - mz^4 + q^2z^6)} \right). \quad (3.3)$$

The constant  $N^2$  is arbitrary which specifies the speed of light of the boundary gauge theory and we choose it to be unity. As a result at the boundary, where  $z \rightarrow 0$ ,

$$f(z) \rightarrow \frac{1}{N^2}, \quad N^2 = \frac{1}{2} \left( 1 + \sqrt{1 - 4\lambda} \right). \quad (3.4)$$

Beyond  $\lambda \leq \frac{1}{4}$  there is no vacuum AdS solution and one cannot have a conformal field theory at the boundary. Casualty leads to new bounds on  $\lambda$  [22, 23]. The drag force on a moving heavy quark and the jet quenching parameter in the background of  $RNAdS$  black hole in Gauss-Bonnet gravity was studied in [24]. By setting Gauss-Bonnet coupling to be zero, the analytic solution for drag force in the case of Reissner-Nordström-AdS background was found.

We intend to study the effect of the higher derivative corrections to the heavy quark potential (2.11) and the inter-quark distance (2.6). We cannot solve (2.11) and (2.6) analytically and we have to resort to numerical methods. For different values of  $\lambda = 0.04, 0.08$  and  $0.12$ , the binding energy of the heavy meson versus the inter-quark distance is plotted in the Fig. 4. It is clearly seen that at a given inter-quark distance by increasing the  $\lambda$  the binding energy also increases. This is an interesting phenomena, because it implies that as the coupling constant increases the U-shaped of the connected string may be discount. We leave physics of this phenomena for further investigation.

At a given energy, the increase in the  $\lambda$  leads to a decrease in the  $r_d$ . This confirms that considering the higher derivative corrections in the gravity background leads to a decrease

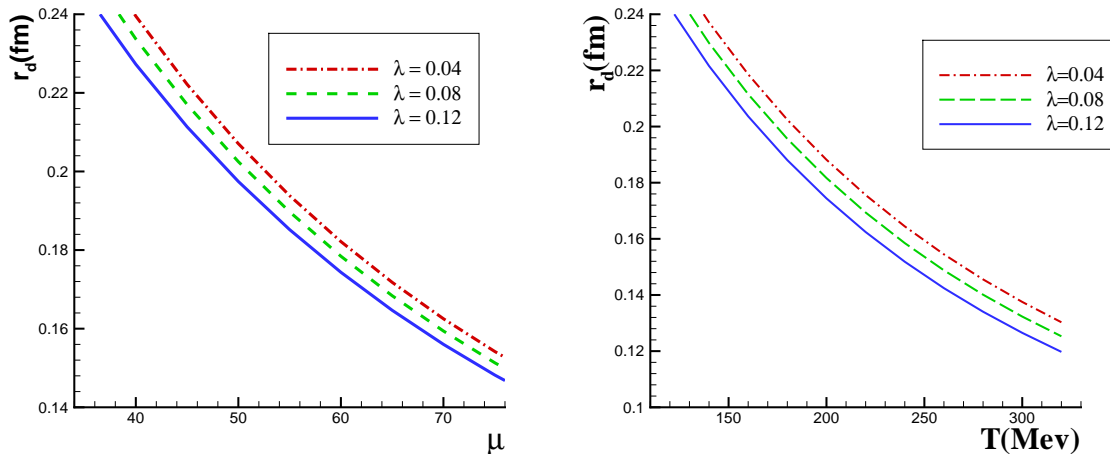


Figure 5: Left: The inter-quark distance versus the chemical potential in the quark-gluon plasma at fixed temperature  $T = 100\text{MeV}$  and  $n = 1$ . Right: The inter-quark distance versus the temperature in the quark-gluon plasma at fixed  $\mu = 30\text{MeV}$  and  $n = 1$ .

in the dissociation length of heavy meson in the boundary gauge theory [20]. However in our study, the boundary gauge theory consists of the light and heavy quarks. Then one can extend the results of [20] to the medium quark, too. This result has been observed for the first time in this study. To more clarify this observation, we plot the dissociation length  $r_d$  versus the chemical potential and the temperature of the hot plasma in Fig. 5. In this figure, we have fixed the coupling constant as  $\lambda = 0.04, .08$  and  $0.12$ . One finds from this figure that at a given temperature or chemical potential, by increasing the  $\lambda$ ,  $r_d$  becomes shorter. The current lattice results show the surprising resistance to the melting of the 1S states such as  $J/\psi$ ,  $\eta_c$  at least up to  $T > 1.5T_c$ . The  $\psi'(2S)$  and  $\chi_c(1P)$ , however, do melt right above  $T_c$  even in the lattice determination [9]. One finds that by considering higher derivative corrections the heavy meson in the medium composed of light quarks and gluons will melt at higher temperature.

## 4 The rotating heavy meson in the quark medium

It is desirable to take into account the nonzero angular momentum of the quark antiquark pair in our study. Excited  $q - \bar{q}$  pair could have the non-zero angular momentum and spin. A holographic picture of the melting process of low spin mesons in the quark-gluon plasma was discussed in [15]. The large spin case was also studied in [16].

In this section, we are going to obtain some information for an excited heavy  $q - \bar{q}$  pair in plasma with a non-zero chemical potential from the gravity dual theory. We consider the

rotating heavy meson in the quark gluon plasma phase of QCD and investigate melting of it. We expect that excited heavy mesons like  $\psi'(2S)$  and  $\chi_c(1P)$  have a larger dissociation length than the ground state mesons like  $J/\psi$ . It is shown that by increasing the spin of heavy meson, the dissociation length also increases which confirms our expectation. This subject was also studied in [16, 18, 25]. Here, we discuss the inter-quark distance in the presence of the chemical potential.

We consider a rotating meson on the  $\rho, \theta$  plane with  $x_3$  the direction perpendicular to the plane of rotation. Then the space time metric in (2.2), will be as

$$ds^2 = \frac{R^2}{z^2} \left( f(z) dt^2 + d\rho^2 + \rho^2 d\theta^2 + dx_3^2 + \frac{dz^2}{f(z)} \right), \quad (4.1)$$

where our four-dimensional space is along  $t, \rho, \theta$  and  $x_3$ . In order to describe the rotation of a quark-antiquark pair the end-points of the string on the probe brane must satisfy the Neumann boundary conditions. To study a rotating string, we make use of the Nambu-Goto action in the above background given by

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det g_{ab}}. \quad (4.2)$$

The coordinates  $(\sigma, \tau)$  parameterize the induced metric  $g_{ab}$  on the string world-sheet. Indices  $a, b$  run over the two dimensions of the world-sheet. Let  $X^\mu(\sigma, \tau)$  be a map from the string world-sheet into spacetime and let us define  $\dot{X} = \partial_\tau X$ ,  $X' = \partial_\sigma X$ , and  $V \cdot W = V^\mu W^\nu G_{\mu\nu}$  where  $G_{\mu\nu}$  is the AdS black hole metric. Indices  $\mu, \nu$  run over the five dimensions of spacetime. Then

$$-g = -\det g_{ab} = (\dot{X} \cdot X')^2 - (X')^2 (\dot{X})^2. \quad (4.3)$$

We choose to parameterize the two-dimensional world-sheet of the rotating string  $X^\mu(\sigma, \tau)$  according to

$$X^\mu(\sigma, \tau) = (t = \tau, \quad \rho = \sigma, \quad z = z(\rho), \quad \theta = \omega t). \quad (4.4)$$

Simply, what this means is that the radius of the rotating quark-antiquark on the probe brane changes with the fifth direction of the bulk space as we move further into the bulk. In arriving at the parametrization (4.4), we made use of the fact that the quark-antiquark pair is in circular motion with radius  $d$  at a constant angular velocity  $\omega$ . Also, we assumed that the system retains its constant circular motion at all times. Furthermore, the ansatz (4.4) does not show any dragging effects which frees us from applying a force to maintain the rigid rotation [16].

According to our ansatz (4.4), the Nambu-Goto action becomes

$$S = -\frac{R^2}{2\pi\alpha'} \int dt d\rho \frac{1}{z^2} \sqrt{\left(1 - mz^4 + q^2 z^6 - \rho^2 \omega^2\right) \left(\frac{z'^2}{1 - mz^4 + q^2 z^6} + 1\right)}, \quad (4.5)$$

where prime is the derivative with respect to  $\rho$ . It is evident that positivity of the square root in (4.5) requires that  $1 - mz^4 + q^2z^6 - \rho^2\omega^2 \geq 0$ . In the case of zero finite density, it can be verified that all rotating strings with different angular velocity do satisfy this condition [25]. As discussed by [29] at this radial coordinate a horizon develops on the world-volume. The stochastic trailing string and Langevin dynamics was studied in [30]. It was shown that the stochasticity arises at the string world-sheet horizon, and thus is causally disconnected from the black hole horizon in the space-time metric. They conclude that this hints at the non-thermal nature of the fluctuations. It would be interesting to extend the results to the case of finite density, *i.e.*  $RNAdS$  black hole. We leave this problem for further work.

We define lagrangian density as

$$\mathcal{L} = \frac{1}{z^2} \sqrt{\left(1 - mz^4 + q^2z^6 - \rho^2\omega^2\right) \left(\frac{z'^2}{1 - mz^4 + q^2z^6} + 1\right)}, \quad (4.6)$$

and the equation of motion for  $z$  is given by

$$\partial_\rho \left( \frac{z' \sqrt{\left(1 - mz^4 + q^2z^6 - \rho^2\omega^2\right) \left(\frac{z'^2}{1 - mz^4 + q^2z^6} + 1\right)}}{z^2 (z'^2 + 1 - mz^4 + q^2z^6)} \right) - \frac{\partial \mathcal{L}}{\partial z} = 0, \quad (4.7)$$

where  $\frac{\partial \mathcal{L}}{\partial z}$  is easily found from (4.6). The equation of motion for  $z(\rho)$  in (4.7) is nonlinear and coupled. However, for generic values of  $\omega$  we cannot solve this equation analytically and we have to resort to numerical methods. The boundary conditions which solve (4.7) physically means that string terminates orthogonally on the brane in the boundary which in turn implies Neumann boundary conditions. Also  $z'(0) = 0$  has been considered at the tip of the string where  $\rho = 0$ . To check the validity of our solutions, we choose  $\rho = d$  so that at  $\rho = 0$  we keep the condition  $z'(0) = 0$ . This system was studied in details in [25]. It was argued that as  $\omega$  decreases the string endpoints become more and more separated, *i.e.* the radius of the open string at the boundary increases and it penetrates deeper into the horizon. We also find the same physics in our case. The constant of the motion is angular momentum  $J$  which can be found as

$$J = \frac{\rho^2\omega}{z^2} \sqrt{\frac{1 + z'^2/f(z)}{f(z) - \rho^2\omega^2}} \quad (4.8)$$

At the tip of the string, we have

$$z = z_0, \quad \rho = 0. \quad (4.9)$$

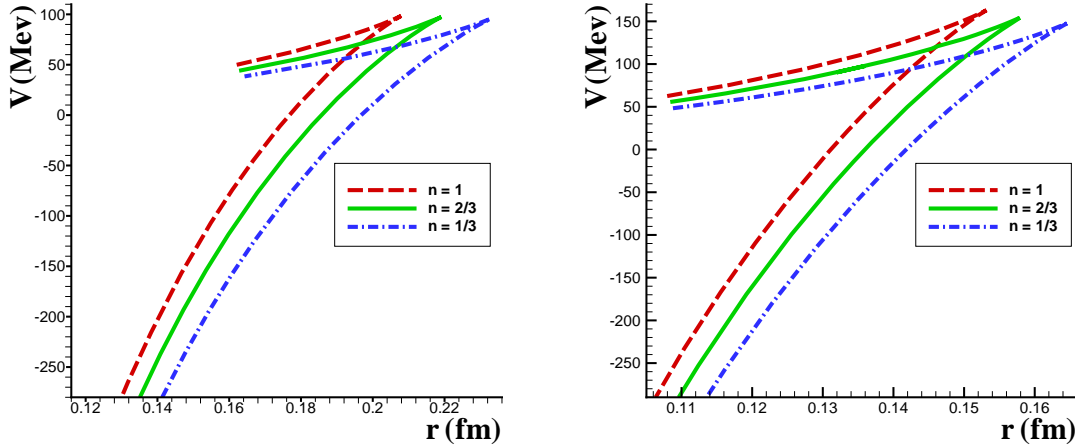


Figure 6: Left: The binding energy of a rotating heavy meson versus the inter-quark distance in the quark-gluon plasma at fixed  $\mu = 30\text{MeV}$  and  $T = 200\text{MeV}$ . Left:  $\omega = 500$ . Right:  $\omega = 1500$ .

It is clear that at the tip of the string  $J = 0$ . This is reasonable, because this is the special point on the string which is static all the times.

It would be interesting to investigate melting of a rotating heavy meson in the quark medium.<sup>6</sup> Then one should study the binding energy of the rotating meson. One finds the standard calculations in [18, 25]. We present the numerical results in the Fig. 6. We have fixed the chemical potential and the temperature as  $\mu = 30\text{MeV}$  and  $T = 200\text{MeV}$ , respectively. One finds that as the angular-momentum  $\omega$  increases, the binding energy also increases. Also as the static case, by increasing the flavor number  $N_f = nN_c$  the inter-quark distance becomes shorter. To more clarify the effect of the  $\omega$  on the inter-quark distance, we plot  $r_d$  versus the  $\omega$  in the left plot of Fig. 7. One concludes that the maximum height of the radius of the quark-antiquark system depends on angular velocity.

One finds from this figure that by decreasing  $\omega$ , the dissociation length becomes larger. What is the relation between angular-momentum and spin of particle? The right plot of Fig. 7 explores this question. The physical part is when by decreasing the angular momentum, the spin of particle increases [16]. The other part is related to the long strings which are not stable. As a result we find that by increasing the spin of heavy meson, the dissociation length also increases which confirms our expectation from lattice. The same physics was discussed in the case of rotating baryons in [28], too.

<sup>6</sup>We restrict our attention to the case of quark-gluon phase.

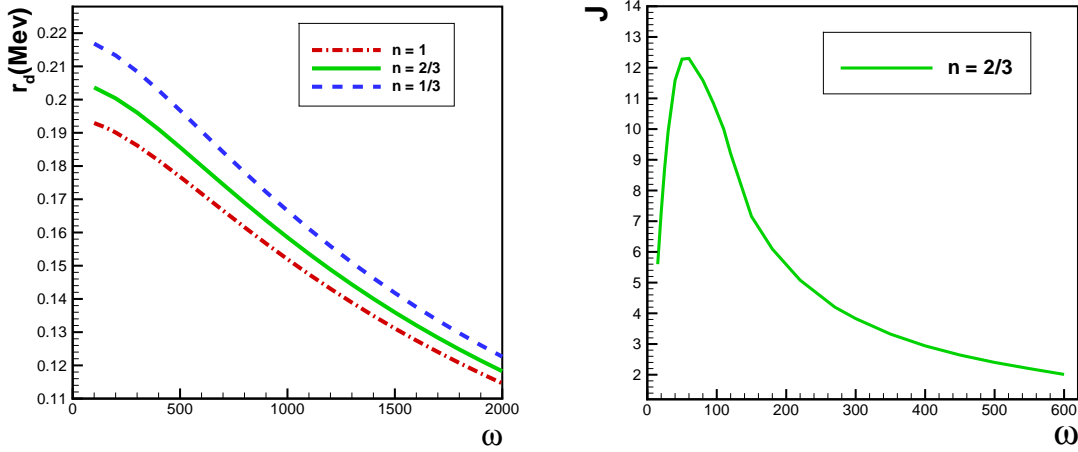


Figure 7: Left: The inter-quark distance versus the angular-momentum in the quark-gluon plasma phase at fixed  $\mu = 30MeV$  and  $T = 200MeV$ . Right: The spin of heavy meson versus the angular-momentum.

## 5 Conclusion

The melting of quarkonium states is of high importance in the physics of QGP at RHIC and LHC. It has long been regarded to be one of the cleanest signatures of plasma formation. In particular, quarkonium states such as the  $J/\psi$  meson are expected to melt in the QGP at higher temperatures than the excited states such as  $\psi'(2S)$  and  $\chi_c(1P)$ .

In this paper we have studied the dissociation length of a heavy meson in the presence of chemical potential. In particular, the inter-quark distance in the static and rotating case was investigated. It was shown that in both phase of QCD, i.e quark-gluon plasma and hadronic phase the dissociation length of quark-antiquark pair becomes shorter as the temperature or the chemical potential increases. We conclude that the melting mechanism in the quark-gluon plasma and in the hadronic phase are the same, *i.e.* the interaction between heavy quarks is screened by the light quarks.

We have considered the static case and extended the results to higher derivative corrections, *i.e.*  $\mathcal{R}^2$  which correspond to finite coupling corrections in the hot plasma. As one expects from the results of [20], the dissociation length becomes shorter as the coupling constant increases. This confirms the claim of [20] and extends it to the medium composed of light quarks and gluons.

To study melting of excited quarkonium states, the dissociation length of a rotating heavy meson in the quark-gluon plasma phase of QCD was also studied. It was shown that by increasing the spin of heavy meson, the dissociation length also increases which confirms

our expectation from Lattice. Also the effect of increasing flavor quarks on the dissociation length was investigated and it was shown that as the flavor quarks increase, the dissociation length decreases.

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## Appendix A:

In this appendix, we calculate the integral in (2.10). We consider the general formula as

$$\int_y^\infty \frac{dx}{\sqrt{(x-a)(x-b)(x-c)}} = gF(\phi, k), \quad (5.1)$$

where

$$g = \frac{2}{\sqrt{a-c}}, \quad k^2 = \frac{b-c}{a-c}, \quad \phi = \text{Sin}^{-1} \sqrt{\frac{a-c}{y-c}}. \quad (5.2)$$

To calculate (2.10), we convert it to the form of (5.1). We change variable  $z$  as

$$z = \frac{1}{\sqrt{x}}. \quad (5.3)$$

Then one should solve this integral

$$\int_{\frac{1}{z_0^2}}^\infty \frac{dx}{\sqrt{-q^2 + mx - x^3}}, \quad (5.4)$$

The parameters of  $a, b$  and  $c$  can be found by solving these equations:

$$a + b + c = 0, \quad abc = -q^2, \quad (ab + ac + bc) = -m. \quad (5.5)$$

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